



## Research Article

# Input–output finite-time stabilization of linear systems with finite-time boundedness



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## ABSTRACT

The paper presents linear system Input–Output Finite-Time Stabilization (IO-FTS) method under Finite-Time Boundedness (FTB) constraint. A state feedback controller is designed, via Linear Matrix Inequalities (LMIs), to guarantee the system both IO-FTS and FTB. The proposed methods are applied to the guidance design of a class of terminal guidance systems to suppress disturbances with IO-FTS method and FTB constraints simultaneously satisfied. The simulation results illustrate the effectiveness of the proposed methods.

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## 1. Introduction

The concept of Finite-Time Stability (FTS) was introduced in 1960s. Up until now, much work has been done in this field. Given a bound on the initial condition, a system is said to be finite-time stable if the state does not exceed a certain threshold during a specified time interval. It worth noting that there is a different notion of FTS [1], which requires the system state to reach the system equilibrium in a finite time interval, and the property is called finite-time attractiveness in some research [2,3]. In the remainder of this paper, FTS we mentioned refers to the former one.

While external disturbances are considered, FTS is extended to Finite-Time Boundedness (FTB). In the light of results coming from Linear Matrix Inequality (LMI) theory, many optimization and control problems can be formulated and solved by using LMIs [4–8]. Sufficient conditions for FTB and finite-time stabilization of linear time-invariant system were provided [9]. Then the FTB problem was studied in many other cases, such as discrete-time linear systems [10], stabilization via dynamic output feedback [11], impulsive dynamical systems [12], linear time-varying systems with jumps [13] and impulsive dynamical linear systems [14].

Recently, necessary and sufficient conditions for finite-time stability of impulsive dynamical linear systems were also proposed [15].

Input–Output Finite-Time Stability (IO-FTS) has been given in [16], which means that, given a class of norm bounded input signals over a specified time interval  $[0, T]$ , the outputs of the system are also norm bounded over  $[0, T]$ . Amato et al. [16] provided methods to solve IO-FTS problem via static state feedback with the disturbance considered as input of the system. Therefore, this contribution can be used to disturbance suppression.

It can be concluded that FTB consider the system state (not exceeding a given threshold) in a finite time interval while IO-FTS only consider the output (norm bounded with input disturbances satisfying some boundedness conditions) in a finite time interval. However, in some kind of practical applications, system state boundedness and disturbance suppression in a finite time interval are both concerned. The FTB only concerns system state not exceeding a given threshold, and cannot take into account some other system performance measures. Hence, by combining FTB and IO-FTS, the behaviors of system state and output can be considered comprehensively, as dealt in this paper.

It should be mentioned that, while considering system behavior in INFINITE time, many disturbance suppression control methods can guarantee asymptotic stability of the systems [17], such as  $H_\infty$  Control and Linear Quadratic Optimal Control [18]. However, because of the independence of FTS and Lyapunov Asymptotic Stability (LAS) (A system can be FTS but not LAS,

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and vice visa. See [9]), finite-time disturbance suppression control methods always do not have a similar capability of stabilizing the system in the finite time interval.

The remainder of this paper is organized as follows. Section 2 presents the basic definitions and the problem statement. Section 3 proposes the sufficient conditions which guarantee both FTB and IO-FTS of a linear time-invariant system. State feedback controller design method is also proposed. Section 4 shows the application of the proposed methods to a class of terminal guidance systems. The conclusion is given in Section 5.

## 2. Problem statement

Consider the following time-invariant linear system

$$\dot{x}(t) = Ax(t) + Gw(t), \quad (1a)$$

$$y(t) = Cx(t), \quad (1b)$$

where  $x(t) \in \mathbb{R}^n$  are the state,  $w(t) \in \mathbb{R}^l$  are exogenous disturbance and satisfies

$$\int_0^T w^T(t)S_1w(t)dt \leq 1. \quad (2a)$$

Matrices  $A \in \mathbb{R}^{n \times n}$ ,  $G \in \mathbb{R}^{n \times l}$ ,  $C \in \mathbb{R}^{m \times n}$  and  $S_1 \in \mathbb{R}^{l \times l}$ .  $S_1$  is a positive definite symmetric matrix. We provide the following definitions, which is slightly different with the ones in [9] (FTB) and [16] (IO-FTS).

**Definition 1 (FTB).** (System (1) is said to be finite-time bounded (FTB) with respect to  $(c_1, c_2, T, R, S_1)$ , with  $c_1 < c_2$ ,  $R > 0$ ,  $T > 0$ , if

$$x^T(0)Rx(0) \leq c_1 \Rightarrow x^T(t)Rx(t) \leq c_2, \quad \forall t \in [0, T].$$

**Definition 2 (IO-FTS).** (Consider zero initial condition ( $x(0) = 0$ ), System (1) is said to be Input–Output Finite-Time Stable (IO-FTS) with respect to  $(T, S_2)$ , with  $T > 0$ ,  $S_2 > 0$  if

$$y^T(t)S_2y(t) \leq 1, \quad \forall t \in [0, T]. \quad (2b)$$

**Remark 1.** In the area of the finite time system area, most of scholars consider the following two classes of exogenous disturbances for the FTB and IO-FTS problems,

$$w(t) \in L_{2,R}[0, T], \quad \int_0^T w^T(t)Rw(t)dt \leq 1$$

$$w(t) \in L_{\infty,R}[0, T], \quad \max_{t \in [0, T]} w^T(t)Rw(t) \leq 1$$

For instance, Amato et al. [9] considers the FTB problem with the disturbance  $w(t) \in L_{\infty,R}[0, T]$  and Amato et al. [16] considers the IO-FTS with both two classes of disturbances. In this paper, we only focus on the first class of disturbance, i.e.

$$w(t) \in L_{2,R}[0, T], \quad \int_0^T w^T(t)Rw(t)dt \leq 1.$$

**Problem 1 (Both FTB and IO-FTS via state feedback).** Consider the system

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t), \quad (3a)$$

$$y(t) = Cx(t), \quad (3b)$$

where  $u(t) \in \mathbb{R}^r$  is the input,  $B \in \mathbb{R}^{n \times r}$ . Consider a state feedback

$$u(t) = Kx(t) \quad (4)$$

where  $K$  is a matrix to be determined later. Applying this controller to the system (3) resulting the following closed-loop system:

$$\dot{x}(t) = \bar{A}x(t) + Gw(t), \quad (5a)$$

$$y(t) = Cx(t). \quad (5b)$$

where  $\bar{A} = A + BK$ . The problem is to find a state feedback controller in the form of (4) such that the closed-loop system (5) is FTB with respect to  $(c_1, c_2, T, R, S_1)$  and IO-FTS with respect to  $(T, S_2)$ .

## 3. Main result

**Lemma 1 (sufficient condition of FTB).** System (1) is FTB with respect to  $(c_1, c_2, T, R, S_1)$  if, letting  $\tilde{Q} = R^{-\frac{1}{2}}QR^{-\frac{1}{2}}$ , there exist a scalar  $\alpha > 0$  and a symmetric positive definite matrix  $Q \in \mathbb{R}^{n \times n}$  such that

$$\begin{bmatrix} A\tilde{Q} + \tilde{Q}A^T - \alpha\tilde{Q} & G \\ G^T & -S_1 \end{bmatrix} < 0, \quad (6a)$$

$$1 + \frac{c_1}{\lambda_{\min}(Q)} < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(Q)}, \quad (6b)$$

where  $\lambda_{\max}(\cdot)$  and  $\lambda_{\min}(\cdot)$  indicate the maximum and minimum eigenvalues of argument, respectively.

**Proof.** Let  $V(x(t)) = x^T(t)\tilde{Q}^{-1}x(t)$ , we have

$$\dot{V} = \begin{bmatrix} x \\ w \end{bmatrix}^T \begin{bmatrix} A^T\tilde{Q}^{-1} + \tilde{Q}^{-1}A & \tilde{Q}^{-1}G \\ G^T\tilde{Q}^{-1} & 0 \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix}. \quad (7)$$

We omit  $(t)$  whenever no ambiguity arises.

Pre and post-multiplying (6a) by

$$\begin{bmatrix} \tilde{Q}^{-1} & 0 \\ 0 & I \end{bmatrix},$$

we obtain

$$\begin{bmatrix} A^T\tilde{Q}^{-1} + \tilde{Q}^{-1}A - \alpha\tilde{Q}^{-1} & \tilde{Q}^{-1}G \\ G^T\tilde{Q}^{-1} & -S_1 \end{bmatrix} < 0. \quad (8)$$

Putting together (7) and (8), we have

$$\dot{V} - \alpha V - w^T S_1 w < 0. \quad (9)$$

Pre and post-multiplying (9) by  $e^{-\alpha t}$ , and integrating from 0 to  $t$ ,  $t \in (0, T]$ , we obtain

$$\int_0^t (e^{-\alpha\tau} \dot{V}) d\tau < \int_0^t e^{-\alpha\tau} w^T S_1 w d\tau.$$

Noting  $\alpha > 0$ , we have

$$e^{-\alpha t} V(x(t)) - V(x(0)) < \int_0^t w^T S_1 w d\tau \leq 1,$$

then

$$V(x(t)) < e^{\alpha t} (1 + V(x(0))),$$

which can be rewritten as

$$x^T(t)R^{1/2}Q^{-1}R^{1/2}x(t) < e^{\alpha t} (1 + x^T(0)R^{1/2}Q^{-1}R^{1/2}x(0)),$$

then

$$\lambda_{\min}(Q^{-1})x^T(t)Rx(t) < e^{\alpha t} (1 + \lambda_{\max}(Q^{-1})x^T(0)Rx(0)). \quad (10)$$

By (6b) and (10), we can obtain, for all  $t \in [0, T]$

$$x^T(t)Rx(t) \leq c_2$$

Therefore, the proof follows.

**Lemma 2 (sufficient condition of IO-FTS).** System (1) is IO-FTS with respect to  $(T, S_2)$  if there exists a symmetric positive definite matrices  $\tilde{Q} \in \mathbb{R}^{n \times n}$  such that

$$\begin{bmatrix} A\tilde{Q} + \tilde{Q}A^T & G \\ G^T & -S_1 \end{bmatrix} < 0, \quad (11a)$$

$$\tilde{Q}^{-1} \geq C^T S_2 C. \quad (11b)$$

**Proof.** Let  $V(x(t)) = x^T(t) \tilde{Q}^{-1} x(t)$ , we have

$$\dot{V} = \begin{bmatrix} x \\ w \end{bmatrix}^T \begin{bmatrix} A^T \tilde{Q}^{-1} + \tilde{Q}^{-1} A & \tilde{Q}^{-1} G \\ G^T \tilde{Q}^{-1} & 0 \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix}. \quad (12)$$

Pre and post-multiplying (11a) by

$$\begin{bmatrix} \tilde{Q}^{-1} & 0 \\ 0 & I \end{bmatrix},$$

we obtain

$$\begin{bmatrix} A^T \tilde{Q}^{-1} + \tilde{Q}^{-1} A & \tilde{Q}^{-1} G \\ G^T \tilde{Q}^{-1} & -S_1 \end{bmatrix} < 0. \quad (13)$$

by (12) and (13), we have

$$\dot{V} - w^T S_1 w < 0,$$

integrating from 0 to  $t$ ,  $t \in (0, T]$ , we obtain

$$V(x(t)) - V(x(0)) < \int_0^t w^T S_1 w d\tau.$$

Noting  $x(0) = 0$ , we have

$$x^T(t) \tilde{Q}^{-1} x(t) < \int_0^t w^T S_1 w d\tau < \int_0^T w^T S_1 w d\tau \leq 1. \quad (14)$$

By (11b), (14) and system (1), we can obtain

$$y^T S_2 y = x^T C^T S_2 C x \leq x^T(t) \tilde{Q}^{-1} x(t) < 1.$$

Therefore, the proof follows.

**Theorem 1 (Both FTB and IO-FTS sufficient condition).** System (1) is both FTB and IO-FTS with respect to  $(c_1, c_2, T, R, S_1, S_2)$  if, letting  $\tilde{Q} = R^{-\frac{1}{2}} Q R^{-\frac{1}{2}}$ , there exist a scalar  $\alpha > 0$  and a symmetric positive definite matrix  $Q \in \mathbb{R}^{n \times n}$  such that

$$\begin{bmatrix} A\tilde{Q} + \tilde{Q}A^T - \alpha\tilde{Q} & G \\ G^T & -S_1 \end{bmatrix} < 0, \quad (15a)$$

$$\tilde{Q}^{-1} \geq e^{\alpha T} C^T S_2 C, \quad (15b)$$

$$1 + \frac{c_1}{\lambda_{\min}(Q)} < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(Q)} \quad (15c)$$

**Proof.** According to Lemma 1, it is obvious that (15a) and (15c) guarantee system (1) FTB with respect to  $(c_1, c_2, T, R, S_1)$ . We will demonstrate that (15a) and (15b) guarantee system (1) IO-FTS with respect to  $(T, S_2)$ .

Let  $V(x(t)) = x^T(t) \tilde{Q}^{-1} x(t)$ , we have

$$\dot{V} = \begin{bmatrix} x \\ w \end{bmatrix}^T \begin{bmatrix} A^T \tilde{Q}^{-1} + \tilde{Q}^{-1} A & \tilde{Q}^{-1} G \\ G^T \tilde{Q}^{-1} & 0 \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix}. \quad (16)$$

Pre and post-multiplying (15a) by

$$\begin{bmatrix} \tilde{Q}^{-1} & 0 \\ 0 & I \end{bmatrix},$$

we obtain

$$\begin{bmatrix} A^T \tilde{Q}^{-1} + \tilde{Q}^{-1} A - \alpha \tilde{Q}^{-1} & \tilde{Q}^{-1} G \\ G^T \tilde{Q}^{-1} & -S_1 \end{bmatrix} < 0. \quad (17)$$

By (16) and (17), we have

$$\dot{V} - \alpha V - w^T S_1 w < 0, \quad (18)$$

pre and post-multiplying (18) by  $e^{-\alpha t}$ , and integrating from 0 to  $t$ ,  $t \in (0, T]$ , we obtain

$$\int_0^t (e^{-\alpha \tau} V)' d\tau < \int_0^t e^{-\alpha \tau} w^T S_1 w d\tau.$$

Noting  $\alpha > 0$  and  $x(0) = 0$ , we have

$$e^{-\alpha t} V(x(t)) < \int_0^t w^T S_1 w d\tau \leq 1. \quad (19)$$

By (15b), (19) and system (1), we can obtain

$$y^T S_2 y = x^T C^T S_2 C x \leq e^{-\alpha T} x^T(t) \tilde{Q}^{-1} x(t) \leq e^{-\alpha T} x^T(t) \tilde{Q}^{-1} x(t) < 1$$

Therefore, the proof follows.

**Corollary 1.** Letting  $\tilde{Q} = R^{-\frac{1}{2}} Q R^{-\frac{1}{2}}$ , if there exist a scalar  $\alpha > 0$ , a symmetric positive definite matrix  $\tilde{Q} \in \mathbb{R}^{n \times n}$  and matrix  $L \in \mathbb{R}^{T \times n}$  such that

$$\begin{bmatrix} A\tilde{Q} + \tilde{Q}A^T + BL + L^T B^T - \alpha\tilde{Q} & G \\ G^T & -S_1 \end{bmatrix} < 0, \quad (20a)$$

$$\tilde{Q}^{-1} \geq e^{\alpha T} C^T S_2 C, \quad (20b)$$

$$1 + \frac{c_1}{\lambda_{\min}(Q)} < \frac{c_2 e^{-\alpha T}}{\lambda_{\max}(Q)} \quad (20c)$$

then the Problem 1 can be solved by  $K = L\tilde{Q}$ .

**Proof.** Replacing  $A$  in (15a) with  $\bar{A}$ , letting  $L = K\tilde{Q}$ , then (20a) holds. According to Theorem 1, the proof follows.

**Remark 2.** IO-FTS is defined under zero initial condition ( $x(0) = 0$ ). The initial condition and input both influence the system state and output. For linear system

$$\dot{x}(t) = Ax(t) + Bu(t),$$

$$y(t) = Cx(t),$$

we have the solution

$$x(t) = e^{At} x(0) + \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau,$$

$$y(t) = Ce^{At} x(0) + C \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau. \quad (21)$$

Therefore, from (21), it is concluded that the influences from initial condition  $x(0)$  and input  $u(t)$  to output  $y(t)$  are independent. So the controllers designed in zero initial condition or non-zero initial condition have the similar effect, although (2b) may be not satisfied by closed-loop system while  $x(0) \neq 0$ .

#### 4. Examples

Two examples are presented so as to illustrate the applicability of the proposed results.

**Example 1.** Consider the linear system (3) defined by

$$A = \begin{bmatrix} 1.5 & 1 \\ 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

According to Corollary 1, using the parameters in Table 1, the controller gain  $K$  of both FTB and IO-FTS can be obtained. For comparison, we also give the  $K$ s guaranteeing FTB-Only and guaranteeing IO-FTS-Only, respectively. Results are as follows.

Both FTB and IO-FTS:  $K = [-3.1760 \quad -24.8640]$ ,

FTB-Only:  $K = [-0.7620 \quad -8.2520]$ ,

IO-FTS-Only:  $K = [-14.0376 \quad -65.9180]$ .

Figs. 1 and 2 present the comparisons of their effects.

In Fig. 1, it can be shown that IO-FTS-Only Method is the most effective to drive the output close to zero. However, Fig. 2 shows that the state measure  $x^T(t)Rx(t)$  of IO-FTS-Only method exceed the constraint defined by  $c_2$ . For FTB-Only method and the proposed method (both FTB and IO-FTS), it can be seen that the

**Table 1**  
Simulation parameters.

$\alpha$	$c_1$	$c_2$	$T$	$S_1$	$S_2$	$R$	$x(0)$
0.1	2	3	3	$10^{-4}I$	$0.07I$	$\text{Diag}(0.1, 10)$	$(-3 \ 0.2)^T$

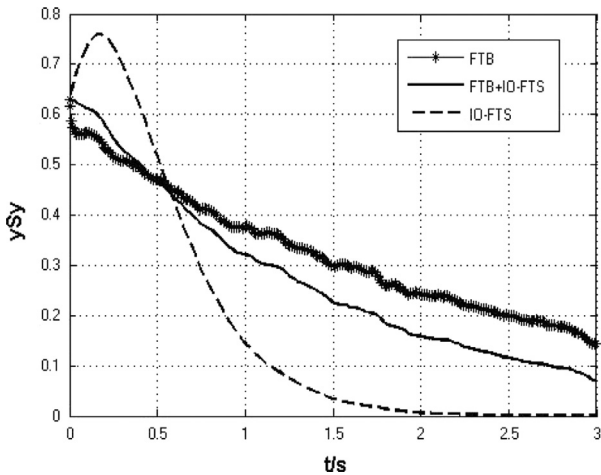


Fig. 1. Comparison of the  $y^T S_2 y$ .

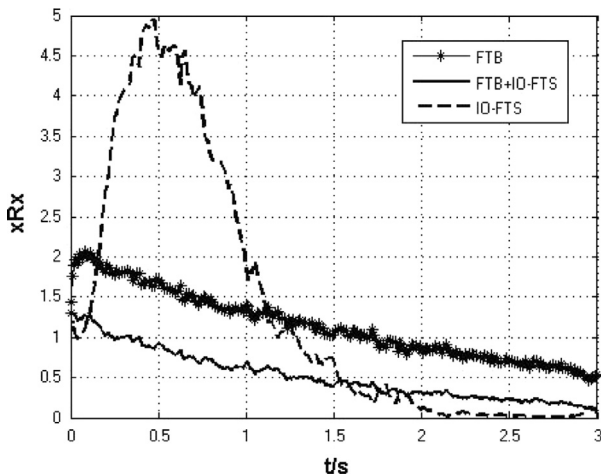


Fig. 2. Comparison of the state measure  $x^T R x$ .

proposed method has better disturbance suppression effect. It can be concluded that the proposed method (both FTB and IO-FTS) can keep the state measure  $x^T(t)Rx(t)$  in the limited range of  $c_2$ , and drive the output tends to zero in the finite time interval.

**Example 2.** Many guidance applications of control theory are given in early researches [19,20]. As a typical finite-time system, a terminal guidance scenario [21] is presented in Fig. 3.

where  $l$  is defined as the perpendicular deviation between the missile and the target;  $r$  is the distance between the missile and the target;  $v_M, v_T$  are the velocities of the missile and target, respectively;  $a_M, a_T$  are the maneuver accelerations of the missile and target, whose directions are vertical to  $v_M$  and  $v_T$ , respectively;  $\varphi_M, \varphi_T$  are the angles of the velocity directions with respect to the reference line  $ox$ .

The terminal guidance scenario can be simplified to satisfy the following assumptions [21]:

- 1) The relative engagement between the missile and the target is approximately considered as head-on form, which means the angles  $\varphi_M$  and  $\varphi_T$  are small.
- 2) Closing velocity  $\dot{r}$  is a constant.

Under these assumptions, we have

$$\ddot{l} = a_M \cos \varphi_M - a_T \cos \varphi_T = a_M - a_T. \quad (22)$$

Define  $a_{iC}, i=M, T$  as acceleration commands,  $\tau_i, i=M, T$  as time constants of acceleration responses of the missile and target, respectively. Then we have

$$\dot{a}_i = (a_{iC} - a_i) / \tau_i, \quad i=M, T, \quad (23)$$

which means that the missile and target need time  $\tau_i, i=M, T$  to adjust from current acceleration  $a_i, i=M, T$  to expected acceleration  $a_{iC}, i=M, T$ .

By (22) and (23), we can obtain a state space description of the system

$$\begin{bmatrix} \dot{l} \\ \ddot{l} \\ \dot{a}_M \\ \dot{a}_T \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -\frac{1}{\tau_M} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau_T} \end{bmatrix} \begin{bmatrix} l \\ \dot{l} \\ a_M \\ a_T \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_M} \\ 0 \end{bmatrix} a_{MC} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\tau_T} \end{bmatrix} a_{TC}, \quad (24a)$$

$$y = l, \quad (24b)$$

where  $y$  is the output, whose value at the final time of guidance can be regarded as the miss distance of the guidance. Since the terminal guidance is a typical finite-time scenario, the model describe the control process in a finite-time interval  $[0, T]$ , and

$$T = r(0)/V_c.$$

where  $V_c = \dot{r}$  is the closing velocity, a constant according to assumption 2.

The target maneuver acceleration command  $a_{TC}$  is considered as an external disturbance, state feedback is used to design

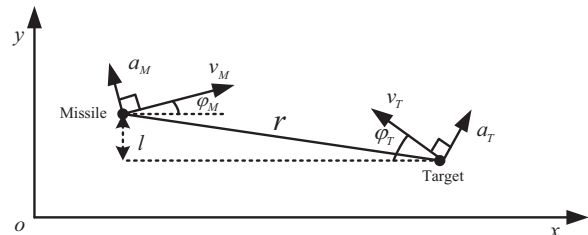
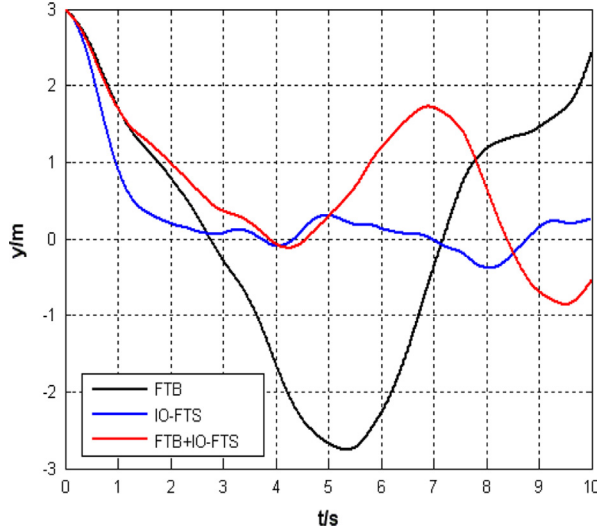


Fig. 3. Terminal guidance scenario geometry.

**Table 2**  
Simulation parameters.

$\tau_M$	$\tau_T$	$\alpha$	$c_1$	$c_2$	$T$	$S_1$	$S_2$	$R$	$x(0)$
0.25	0.25	0.11	4	10	10	$10^{-4}I$	$0.03I$	Diag(0.1, 0.1, 1, 0.1)	$(3 \ -0.5 \ -1 \ 2)^T$



**Fig. 4.** Comparison of the output  $y$ .

controller with the objective to minimize the impact of the disturbance to the output (finite-time disturbance suppression), and some indices not exceeding the physical constraints, such as missile acceleration and seeker's view field (finite-time state bounded). In addition, due to the midcourse guidance error, the terminal guidance system is often with non-zero initial conditions, therefore, the proposed method is just suitable for this problem.

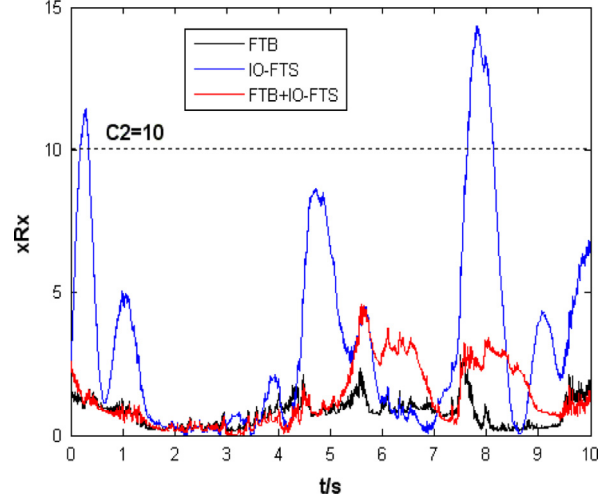
The simulation parameters are given in Table 2,  $\tau_M = \tau_T$  means that missile and target have the same acceleration response capabilities.  $\alpha, c_1, c_2$  is the parameters determined by the initial conditions and the acceleration constraints.  $S_1$  is determined by the amplitude of the target maneuver.  $S_1$  the design requirements of the guidance miss distance.  $R$  is the weighting matrix of the system states.  $x(0)$  is the initial conditions.

According to Corollary 1, using the parameters in Table 2, the controller gain  $K$  of both FTB and IO-FTS can be obtained. For comparison, we also give the  $K$ s guaranteeing FTB Only and guaranteeing IO-FTS Only, respectively. Results are as follows.

Both FTB and IO-FTS:  $K = [-0.9972 \ -1.7232 \ -0.90530 \ 3809]$ ,  
 FTB-Only:  $K = [-0.0003 \ 15.3408 \ -61.9795 \ -0.0537]$ ,  
 IO-FTS-Only:  $K = [2.4883 \ -2.4461 \ 0.2850 \ 0.4875]$ .

Figs. 2 and 3 present the comparisons of their effects.

Fig. 4 shows that the FTB-Only method cannot drive the output close to zero in the finite time horizon, in fact, if  $t$  tends to infinity, the  $y$  of the FTB-Only method (black line) will also tend to infinity. By comparison, the disturbance suppression effect of the IO-FTS-Only method is best, and the Both-FTB-and-IO-FTS method is better than FTB-Only method. However, in Fig. 5, it can be shown that state measure  $x^T(t)Rx(t)$  of IO-FTS Only (blue line) is significantly larger than the other ones and exceeds  $c_2$ , which means it is unacceptable in practical application, since some variables may exceed the physical restraints. It can be concluded that proposed method (both FTB and IO-FTS, red line) can keep the state measure



**Fig. 5.** Comparison of the state measure  $x^T Rx$ . (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

$x^T(t)Rx(t)$  in the limited range of  $c_2$ , and drive the output tends to zero in the finite time interval.

## 5. Conclusion

In this paper we combine finite-time stabilization method with IO-FTS for the design of finite-time stability controller, by which the closed-loop system satisfies both FTB and some input–output constraints. Sufficient conditions guaranteeing both FTB and IO-FTS are proposed, then a state feedback controller can be obtained via LMIs. The methods are applied to guidance design for a class of terminal guidance systems to solve the problem of disturbance suppression under state constraint and non-zero initial conditions. Performance comparisons of the proposed method, IO-FTS-Only method and FTB-Only method are provided at last. The results show that the proposed method combines the characteristics of the other two methods, and is suitable for solving the problem of disturbance suppression with system state bounded in a finite-time interval. The method can be regarded as a finite-time stabilization method considering some finite-time performance constraints.

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